

# MUTUAL INDUCTANCE PROBLEM FOR A SYSTEM CONSISTING OF A CURRENT SHEET AND A THIN METAL PLATE

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## INTRODUCTION

Rapid inspection of aircraft structures for flaws is of vital importance to the commercial and defense aircraft industry. In particular, inspecting thin aluminum structures for flaws is the focus of a large scale R&D effort in the nondestructive evaluation (NDE) community. Traditional eddy current methods used today are effective, but require long inspection times. New electromagnetic techniques which monitor the normal component of the magnetic field above a sample due to a sheet of current as the excitation, seem to be promising. This paper is an attempt to understand and analyze the magnetic field distribution due to a current sheet above an aluminum test sample. A simple theoretical model, coupled with a two dimensional finite element model (FEM) and experimental data will be presented in the next few sections.

A current sheet above a conducting sample generates eddy currents in the material, while a sensor above the current sheet or in between the two plates monitors the normal component of the magnetic field [1,2]. A rivet or a surface flaw near a rivet in an aircraft aluminum skin will disturb the magnetic field, which is imaged by the sensor.

Initial results [2] showed a strong dependence of the flaw induced normal magnetic field strength on the thickness and conductivity of the current-sheet that could not be accounted for by skin depth attenuation alone. It was believed that the eddy current imaging method explained the dependence of the thickness and conductivity of the flaw induced normal magnetic field. Further investigation, suggested the complexity associated with the mutual inductance of the system needed to be studied. The next section gives an analytical model to better understand the phenomenon.

## ANALYTICAL MODEL

The problem is too complex to obtain an exact solution, so a simplified model was created to see what could be learned. A two dimensional two plate problem [2] was analyzed to model the above system. Region 1 modeled the conducting sample as an infinite half space. The current sheet was placed on top of this conducting half space, while another infinite conducting half space was placed immediately above the current sheet (Fig. 1.). The underlying phenomenon is governed by the quasi-static form of Maxwell's equations, with the displacement current neglected. Governing equation in the two conducting regions is the diffusion equation, while the fields are governed by Laplace's equation in air. The equations in region I, II and III respectively are :

$$\nabla^2 A_1 = j\omega\mu\sigma_1 A_1 \quad (1)$$

$$\nabla^2 A_2 = 0 \quad (2)$$

$$\nabla^2 A_3 = j\omega\mu\sigma_3 A_3 \quad (3)$$

$\sigma_1, \sigma_3$  is the conductivity of the two half spaces, while the permeability is  $\mu$  in all regions.  $\omega$  is the angular frequency and  $A_1, A_2$  and  $A_3$  are the sinusoidal steady state RMS magnetic vector potential. in the three regions respectively. The current density is given by

$$J = J_0 e^{j\omega t} \quad (4)$$

where  $J_0$  is the steady state RMS surface value.

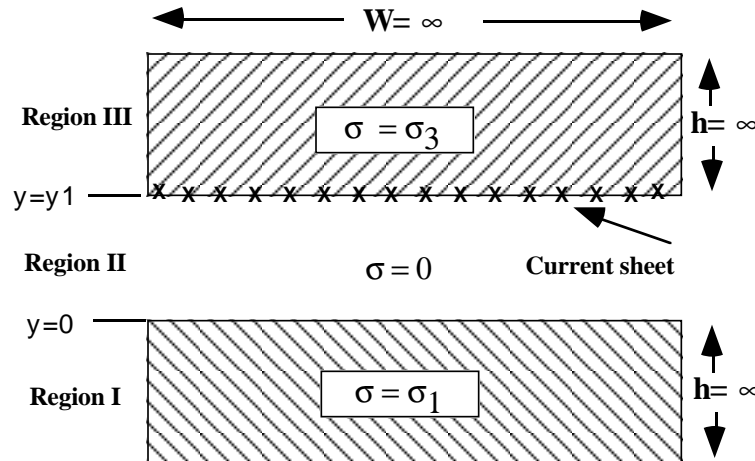


Fig. 1. Analytical model

The boundary conditions in the different regions are:

$$y = 0 \quad A_1 = A_2 \quad \frac{\partial A_2}{\partial y} = \frac{\partial A_1}{\partial y} \quad (5)$$

$$y = 0 \quad A_2 = A_3 \quad \frac{\partial A_2}{\partial y} - \frac{\partial A_3}{\partial y} = \mu J \quad (6)$$

$$y = \infty \quad A_3 = 0 \quad (7)$$

$$y = -\infty \quad A_1 = 0 \quad (8)$$

Applying these boundary conditions, the solutions in the three regions are respectively,

$$A_1 = ae^{k_1 y} \quad (9)$$

$$A_2 = by + c \quad (10)$$

$$A_3 = de^{-k_3 y} \quad (11)$$

where  $k_1 = (i+j)\sqrt{\pi f \mu \sigma_1}$ ,  $k_3 = (i+j)\sqrt{\pi f \mu \sigma_3}$  and the constants are

$$a = b / k_1 \quad (12)$$

$$c = a \quad (13)$$

$$d = \frac{y_1 e^{k_3 y_1}}{y_1 k_3 + 1} (\mu J - \frac{c}{y_1}) \quad (14)$$

$$b = \frac{\mu J}{(1 + \sqrt{\frac{\sigma_3}{\sigma_1}} + y_1 \sqrt{\pi f \mu \sigma_3}) + j \sqrt{\pi f \mu \sigma_3} y_1} \quad (15)$$

The solution clearly indicates the dependence of the magnetic field strength above the sample to be a function of the current sheet thickness, material properties, frequency, and the distance between the sheet and the sample. The magnetic field in the air gap between the two half spaces is proportional to  $b$ , which shows that the field decreases as  $\sigma_3$  increases for a constant  $\sigma_1$ . Practically this would correspond to decreasing the conductivity of the current sheet to obtain higher field levels in the sensor coils.

## FINITE ELEMENT MODEL

To obtain a better understanding of how the above results are related to the problem a finite element (FE) model was constructed. A two dimensional FE model of the problem provided some interesting observations. The model is given in Fig. 2. with the appropriate dimensions. The typical procedure in finite elements is to discretize the region of interest, set up an energy functional for the underlying differential equation to be solved, minimize this functional and finally solve a set of linear equations. In this model the diffusion equation is solved for the magnetic vector potential.

The energy functional is the sum of the electric and magnetic energy in the system and is given by

$$F = \frac{1}{2} \int_{\text{vol}} \frac{1}{\mu} ((\nabla A)^2 + j\omega\sigma A^2 - J_s A) dv \quad (16)$$

This functional is minimized with respect to the magnetic vector potential  $A$  at all the node points in the region to obtain a set of linear equations given by

$$[S] \{A\} = \{Q\} \quad (17)$$

where  $[S]$  is the complex, symmetric and banded global matrix,  $\{A\}$  is a vector of the unknowns and  $\{Q\}$  is a vector of the boundary conditions. This matrix equation is solved using gaussian elimination.

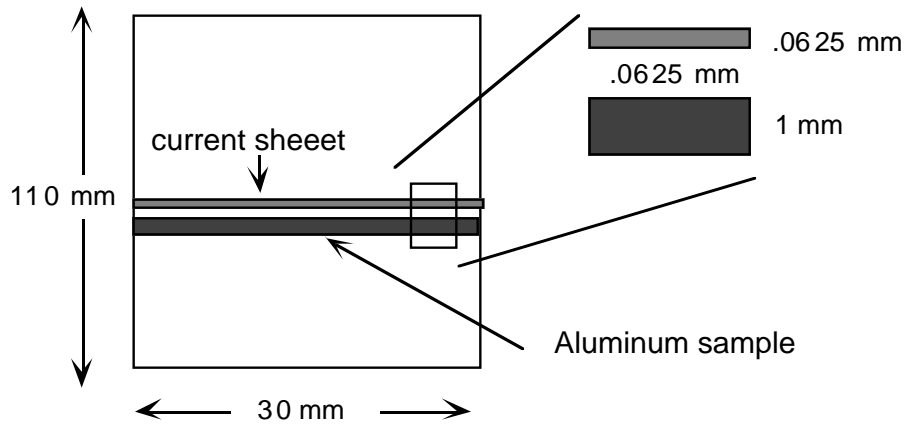


Fig. 2. Finite element model

Fig. 3 is a plot of the magnetic field as a function of the current sheet conductivity in % IACS. This plot confirms that increasing the conductivity of the current sheet reduces the tangential component of the magnetic field levels everywhere. Fig. 4 and Fig. 5 show the magnetic field distribution above and below the current sheet as predicted by the FE model. The vector plot clearly indicates that the field levels are higher for a stainless steel (SS) current sheet than a copper (Cu) current sheet. This is surprising because for a current sheet having a width of .0625 mm, the skin depth decay across the current sheet should be negligible for both SS and Cu. Thus if the field in between the sample and the current sheet were the same, the fields above the sheet should be the same too. Since the field levels are not the same, the model indicates a strong mutual interaction between the current sheet and the sample.

## EXPERIMENTAL OBSERVATIONS

An aluminum sample with a 12 mm diameter hole and a 7 mm fatigue crack was scanned by a pancake coil to monitor the normal component of the magnetic field. A 4.5 Amp sinusoidal ac current at 30 kHz was the source. Details of the experimental setup can be found in an earlier publication [2].

Fig. 6a and Fig. 6b are plots of the normal component of the magnetic field above the Cu current sheet and in between the current sheet and the sample for a fatigue crack. Similarly Fig. 7a and Fig. 7b are plots for a SS current sheet. It is clear that the field levels above the current sheet are much lower than that in between for both the SS and Cu current sheets. Also for a 775  $\mu$  (31 mils) SS current sheet the field levels are about four times higher than the 250  $\mu$  (10 mils) Cu sheet.

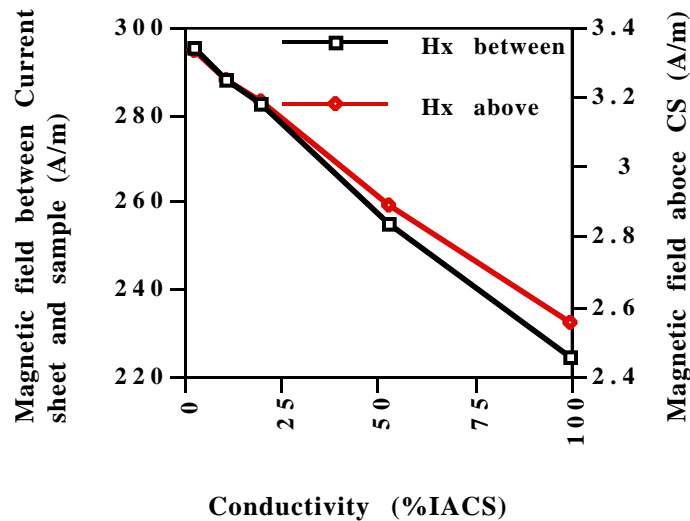


Fig. 3. Finite element prediction of the magnetic field strength for different conducting current sheets.

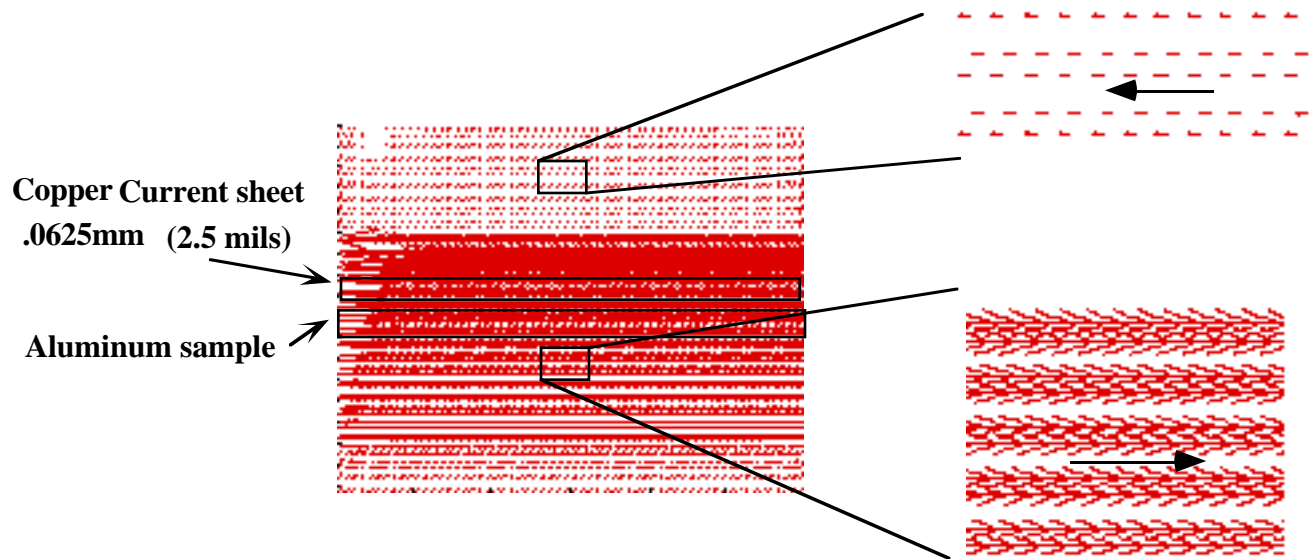


Fig. 4. Finite element prediction of the magnetic field above and below a copper current sheet.

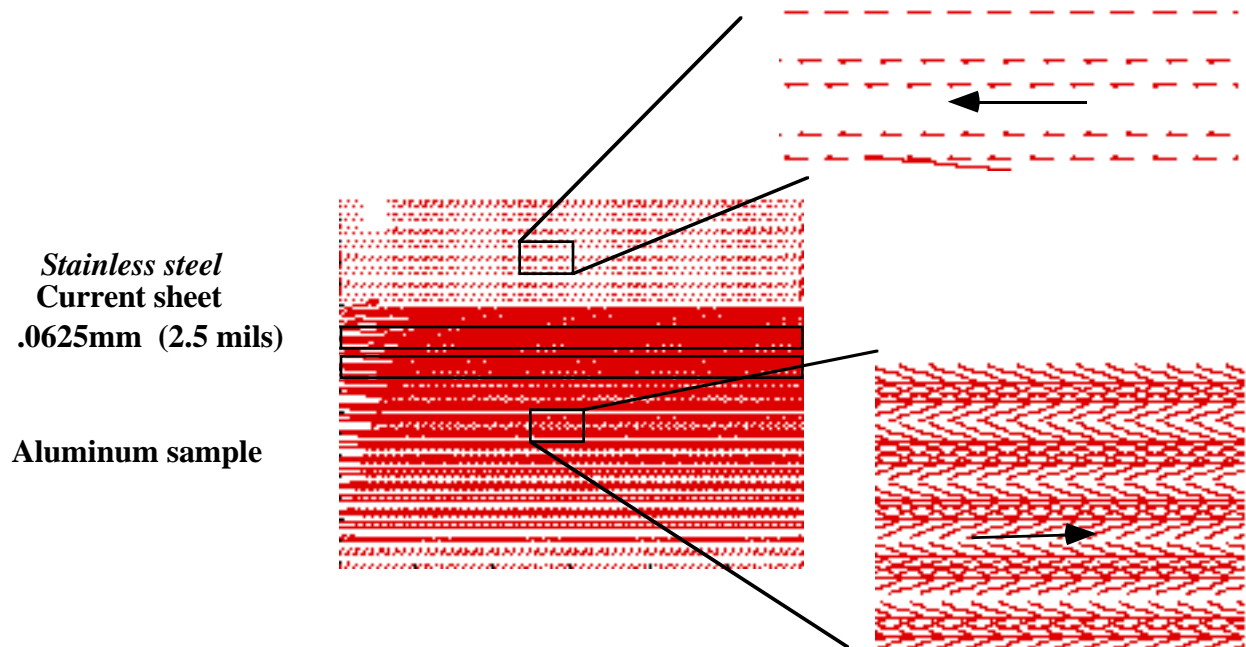


Fig. 5. Finite element prediction of the magnetic field above and below a stainless steel current sheet.

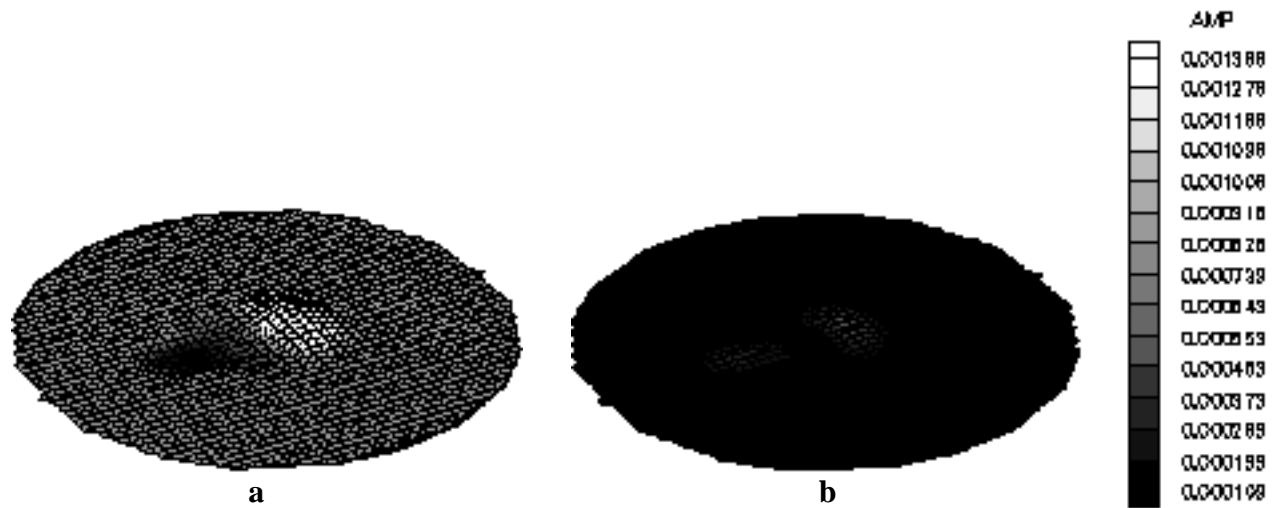


Fig. 6. Normal magnetic field distribution for a fatigue crack in an aluminum sample for a  $250 \mu$  (10 mils) copper current sheet a). in between the two sheets b). above the current sheet.

When the sample is placed beneath the current sheet, a mutual induction effect is developed between the two plates [2]. Since these plates are placed very close to each other (few microns), this effect is very predominant. Also eddy currents are induced in the sample generating a magnetic field that opposes the primary magnetic field. These magnetic fields will induce eddy currents in the current source, which will try to oppose the induced magnetic fields. This mutual effect of the two plates reduces the fields in between as the current sheet thickness increases. Thus, the fields above the current sheet get reduced with a decrease in the conductivity of the current sheet (increase in the resistivity of the current sheet) or with an increase in the thickness of the current sheet. Consequently, if a pickup coil is placed in between, then the field strengths are higher, resulting in a higher pickup coil voltage. Practically this should increase the flaw detection capability in conducting materials.

## SUMMARY

To understand the physics of the phenomenon related to the two plate problem, an analytical and finite element model were developed. The analytical model showed the strong mutual inductance effect between the two plates. This was indicated by the complex dependence of the magnetic field in between the plates on the conductivity of the current sheet. From the finite element model and experimental data, one can conclude that the magnetic field decreases with :

- a) an increase in the conductivity of the current sheet
- b) an increase in the thickness of the current sheet.

Some of the future research includes reducing the distance between the current sheet and the sample, developing tiny pickup coils for measurement of the fields in between the two plates, and using a three dimensional model to look at various defects.

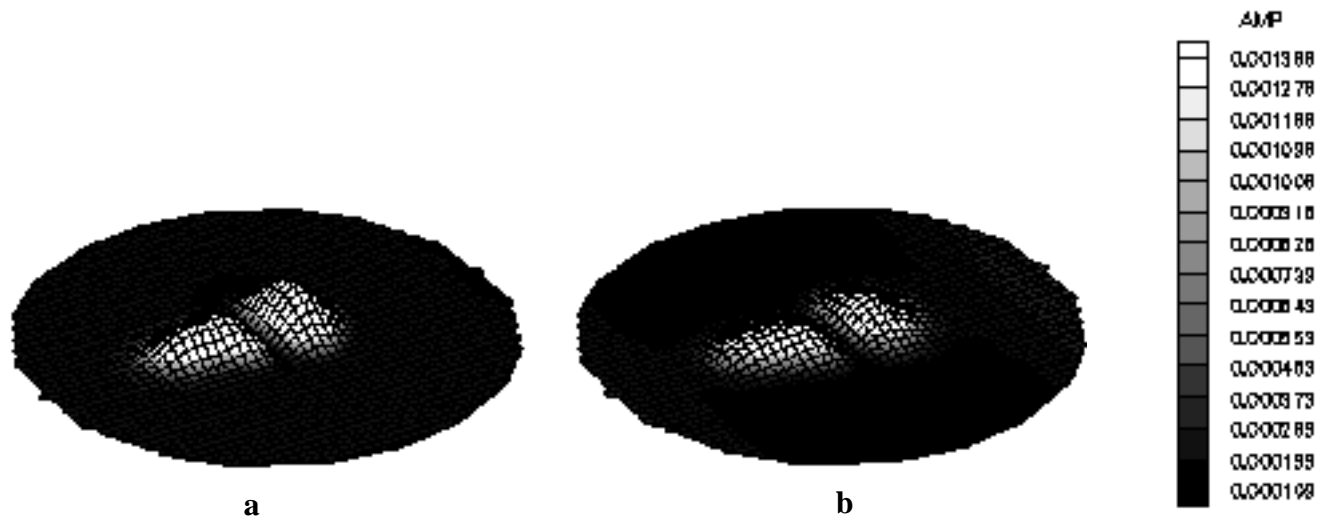


Fig. 7. Normal magnetic field distribution for a fatigue crack in an aluminum sample for a 775  $\mu$  (31 mils) stainless steel current sheet.a). in between the sample and current sheet b). above the current sheet.

#### REFERENCES

1. Sandra Simms, in *Materials Evaluation*, Vol. 51, No. 5, (1993), p. 529.
2. M. Namkung, C.G. Clendenin, J.P. Fulton and B. Wincheski, in *Review of Progress in Quantitative NDE*, Vol. 11A, eds. D.O. Thompson and D.E. Chimenti, (Plenum, New York, 1992) p. 1071.